PB87-196523

Comparison of Three Methods for Calculating the Standard Deviation of the Wind Direction

(U.S.) Environmental Protection Agency
Research Triangle Park, NC

1986

U.S. Department of Commerce
National Technical Information Service
NTIS

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO.	2.	3. RE
EPA/600/J-86/355		PB87-196523

4. TITLE AND SUBTITLE	5. REPORT DATE
COMPARISON OF THREE METHODS FOR CALCULATING THE STANDARD DEVIATION OF THE WIND DIRECTION (Journal Version)	1986
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S)	8. PERFORMING ORGANIZATION REPORT NO.
D. Bruce Turner	
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT NO.
Same as block 12	CDTA1D/08-2182 (FY-85)
	11. CONTRACT/GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS	13. TYPE OF REPORT AND PERIOD COVERED
Atmospheric Sciences Research Laboratory - RTP, NC Office of Research and Development U. S. Environmental Protection Agency Research Triangle Park, NC 27711	14. SPONSORING AGENCY CODE EPA/600/09

15. SUPPLEMENTARY NOTES

REF: J. Climate and Appl. Meteorol., 25(5):703-707, May 1986

16. ABSTRACT

Three methods to calculate wind direction standard deviation are evaluated. Although eight hours of wind data show no significant differences between the methods, synthetically generated data having standard deviations near the maximum possible, show the Yamartino method to perform well. Also, the wind direction for periods, such as an hour, are shown to be well represented by the arctangent of the mean sines and cosines.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group

REPRODUCED BY
U.S. DEPARTMENT OF COMMERCE
NATIONAL TECHNICAL
INFORMATION SERVICE
SPRINGFIELD, VA 22161

18. DISTRIBUTION STATEMENT	19. SECURITY CLASS *(This Report)*	21. NO. OF PAGES
RELEASE TO PUBLIC	UNCLASSIFIED	8
	20. SECURITY CLASS *(This page)*	22. PRICE
	UNCLASSIFIED	PC $9.95/MF $3.50

EPA/600/J-86/355
JOURNAL ARTICLE

Reprinted from JOURNAL OF CLIMATE AND APPLIED METEOROLOGY, Vol. 25, No. 5, May 1986
American Meteorological Society

Comparison of Three Methods for Calculating the Standard Deviation of the Wind Direction

D. BRUCE TURNER*

Meteorology and Assessment Division, Atmospheric Sciences Research Laboratory, Environmental Protection Agency, Research Triangle Park, NC 27711

(Manuscript received 26 July 1985, in final form 23 November 1985)

ABSTRACT

Three methods to calculate wind direction standard deviation are evaluated. Although eight hours of wind data show no significant differences between the methods, synthetically generated data having standard deviations near the maximum possible show the Yamartino method to perform well. Also, the wind direction for periods such as an hour is shown to be well represented by the arctangent of the mean sines and cosines.

1. Introduction

To realistically estimate concentrations from air pollutant sources using air quality simulation models, it is important to represent the atmospheric physics as closely as possible. Therefore, the Meteorology and Assessment Division has advocated the use of fluctuation statistics to estimate atmospheric dispersion (Turner, 1979; Irwin, 1983). Toward this end, it is desirable to calculate σ_a using a method that accurately gives the root-mean-square deviations of the instantaneous wind direction values used.

Since wind direction is a circular function with a crossover point between 360° and 0°, standard statistical methods for linear data sets can not be used directly for calculating the standard deviation of wind azimuth σ_a. Statistical methodology for circular data (Mardia, 1972; Fisher, 1983), relates to the underlying probability density function interpretation of σ_a; this is an interpretation which differs from the meteorologist's definition of σ_a as derived from wind measuring instrumentation, particularly at large σ_a. In this paper, three published methods are evaluated using two types of data sets.

2. Available methods

If all instantaneous values over the time period for which the standard deviation is calculated are retained until the end of the period p, a straightforward method (Irwin, 1980) can be used to account for the crossover. The resultant direction is determined and subtracted from 180°. This adjustment is then added to each individual value and the sum and sum of squares using these values determined again. The standard deviation can then be determined directly from this new sum and sum of squares using Eqs. (1) and (2). Of course,

* Permanent affiliation: National Oceanic and Atmospheric Administration, United States Department of Commerce.

TABLE 1. Data collected and preliminary calculations.

I. Summations of instantaneous values accumulated over the selected time period (for example, 1 hour)

$$\sum \sin a, \quad \sum \cos a, \quad \sum \sin^2 a, \quad \sum \cos^2 a, \quad N, \quad \sum \sin a \cos a$$

II. Calculations made for both Ackermann and Verrall–Williams techniques

$$\overline{\sin a} = \sum \sin a / N, \qquad \overline{\cos a} = \sum \cos a / N, \qquad \bar{a} = \arctan(\overline{\sin a}, \overline{\cos a})$$

$$\sigma_s^2 = \frac{\sum \sin^2 a}{N} - (\overline{\sin a})^2 \qquad \sigma_c^2 = \frac{\sum \cos^2 a}{N} - (\overline{\cos a})^2$$

TABLE 2. Calculations made for Ackermann method.

$$\sigma_{sc} = \frac{\sum \sin a \cos a}{N} - \overline{\sin a}\ \overline{\cos a}$$

$$\sigma_a = \frac{[(\overline{\cos a})^2\sigma_s^2 + (\overline{\sin a})^2\sigma_c^2 - 2\,\overline{\sin a}\ \overline{\cos a}\sigma_{sc}]^{1/2}}{(\overline{\sin a})^2 + (\overline{\cos a})^2}$$

the principal disadvantage of this method is the requirement for intermediate storage of each individual value. A variation of this procedure, but still requiring hundreds of storage locations, has been suggested by Nelson (1984).

Three other methods (Verrall and Williams, 1982; Ackermann, 1983; Yamartino, 1984) overcome these storage requirements provided that the on-line processor can determine the trigonometric functions, sine and cosine, of each instantaneous value. Experimentally, the following five summations are collected over the period p (frequently an hour):

$$\sum \sin a, \quad \sum \cos a, \quad \sum \sin^2 a, \quad \sum \cos^2 a, \quad N,$$

where a is the instantaneous wind direction value and N is the number of instantaneous values used over the time period p. All five sums are needed for the Verrall–Williams approach. In addition to these five values, the Ackermann method also requires $\Sigma \sin a \cos a$. The method of Yamartino requires only $\Sigma \sin a$, $\Sigma \cos a$ and N. Note that calculations discussed here use only the wind direction; the wind speed is not used. Using these summations, the standard deviation of wind azimuth, σ_a, is calculated using the equations given in Tables 1–4.

3. Testing with data from Boulder Atmospheric Observatory tower

Data from sonic anemometers on the Boulder Atmospheric Observatory tower (Kaimal and Gaynor, 1983) were made available to the author. Instantaneous values of u, v, w at 0.1-sec intervals for one hour were available for eight hours as eight data sets. The wind direction was calculated from each u, v pair and corrected for the orientation of the sonic system to yield the wind angle with respect to true north:

$$a = \arctan(u/v) + \text{correction}.$$

The various trigonometric sums discussed in section 2 were then computed.

The distribution of the a's, to the nearest degree, were also tabulated and printed at the conclusion of the processing for each data set to determine if crossover between 360° and 0° would affect the mean and standard deviation calculated by the standard statistical methods:

$$\bar{a} = \sum a/N, \tag{1}$$

$$\sigma_a = [(\sum a^2/N) - \bar{a}^2]^{0.5}. \tag{2}$$

If inspection indicated that the standard statistical computations would be affected, a second run was made adding an appropriate rotation angle to each observation, making the calculation, and then subtracting the rotation from the calculated mean. Four of the eight data sets required the rotation in order to appropriately apply (1–2). FORTRAN statements used to apply the three methods are shown in the Appendix.

Table 5 compares $\bar{a}$ calculated using the equation given in Table 1 (the resultant of the mean sines and cosines) with $\bar{a}$ calculated using equation (1) for the eight data sets. The values from (1) are those obtained after any required rotation to avoid cross-over and therefore are assumed correct. The values are shown to three decimal digits only to allow comparison of the calculational methods, but are not significant to this degree, since it is rare that a wind system can be oriented to yield direction with greater accuracy than about 5°. No difference between the two methods is greater than 0.1°.

Table 6 compares σ_a calculated from (2) with σ_a's estimated using the three methods. The maximum error (Verrall–Williams, data set 1.) is less than 0.5°.

TABLE 3. Calculations made for Verrall–Williams method.

I. The east and north components of vector **1** are

$$E_1 = |\overline{\sin a}| - \sigma_s$$

$$N_1 = |\overline{\cos a}| + \sigma_c$$

II. The east and north components of vector **2** are

$$E_2 = |\overline{\sin a}| + \sigma_s$$

$$N_2 = |\overline{\cos a}| - \sigma_c$$

III. The magnitude of vectors **1** and **2** are

$$M_1 + [E_1^2 + N_1^2]^{1/2}$$

$$M_2 = [E_2^2 + N_2^2]^{1/2}$$

IV. The dot product of vectors **1** and **2** is

$$D = E_1E_2 + N_1N_2$$

$$\sigma_a = 0.5 \arccos[D/(M_1M_2)]$$

TABLE 4. Calculations made for the Yamartino method.

$$\epsilon = \{1. - [(\overline{\sin a})^2 + (\overline{\cos a})^2]\}^{0.5}$$

$$\sigma_a = \arcsin(\epsilon)[1. + 0.1547\epsilon^3]$$

TABLE 5. Average direction and average direction calculated from mean sines and cosines.

Data set	A Average direction $\bar{a}$ (deg)	B $\bar{a}$ calculated using equation in Table I. (deg)	B − A
1	294.469	294.372	−0.097
2	290.602	290.597	−0.005
3	192.851	192.851	same
4	199.854	199.854	same
5	218.304	218.316	0.012
6	29.641	29.650	0.009
7	31.530	31.529	−0.001
8	50.786	50.785	−0.001

This is less than 2%. From the data available in these eight sets, there is little basis for choosing one of the methods over the other.

4. Testing using synthesized data

Yamartino (1984) pointed out that the Ackermann method overestimates σ_a and the Verrall–Williams method underestimates σ_a for values greater than about 30°. The maximum standard deviation from the available eight data sets was less than 22°. Low wind speeds and major wind shifts, such as caused by frontal passages and transition from land to sea breeze and vice versa, contribute to large σ_a values. It has been brought to my attention (Lambeth, personal communication, 1984) that in situations where the surface roughness is large and where there are significant terrain features, mechanical turbulence may cause large directional fluctuations which, in turn, produce large standard deviations. In the extreme, near-uniform distributions around the compass may result. Yamartino also pointed out the occurrence of large σ_a in some of the data to which he had access. It therefore seemed desirable to examine the performance of the three methods on some data sets generated to test these extremes.

Using routines available from the International Mathematical and Statistical Libraries (1982), a number of wind direction distributions were generated with a wide variety of σ_a's. As the distributions were generated, values of wind direction $< 0°$ or $>360°$ were allowed to "wrap around" these crossover points and be included in the distribution. Thus, at the extreme—a distribution with $\sigma_a = 101.7°$—a nearly circular distribution was generated. The same characteristics as noted by Yamartino (overestimation by Ackermann and underestimation by Verrall–Williams) was seen.

Table 7 shows results from a selection of generated distributions. Data for five distributions (each generated with a different "seed") were analyzed for each mean σ_a displayed in the Table. The maximum error seen here for the Yamartino method is less than 1.2° (for $\sigma_a = 76.5$). This is a 1.5% error and verifies Yamartino's statement that his method "is within ±2% of the correct" σ_a.

Some additional tests of the methods were made by taking generated distributions, rotating them (so that the mean would be at different azimuths), and applying the methods. Both the Yamartino and Ackermann methods gave the same σ_a, regardless of the location of the mean; i.e., the methods are insensitive to rotations. However, The Verrall–Williams method gave some variation with rotation of the distribution but no more than 3.6° or 5% of the σ_a.

TABLE 6. Directly calculated σ_a compared with σ_a from three methods.

Data set	A Calculated σ_a Eq. (2) (deg)	B σ_a from Ackermann (deg)	B − A	C σ_a from Verrall–Williams (deg)	C − A	D σ_a from Yamartino (deg)	D − A
1	21.720	21.603	−0.099	21.353	−0.349	21.567	−0.135
2	16.130	15.899	−0.231	15.931	−0.199	16.004	−0.126
3	6.260	6.256	−0.004	6.252	−0.008	6.255	−0.005
4	3.540	3.541	0.001	3.539	−0.001	3.539	−0.001
5	6.020	6.011	−0.009	6.003	−0.017	6.013	−0.007
6	9.191	9.193	0.002	9.171	−0.020	9.177	−0.014
7	4.423	4.425	0.002	4.421	−0.002	4.422	−0.001
8	9.509	9.545	0.036	9.496	−0.013	9.503	−0.006
	Mean Absolute Error:		0.048		0.076		0.037

TABLE 7. Performance of three σ_a methods using generated distributions.

Mean σ_a	Statistic	σ_a method		
		Ackermann	Verrall–Williams	Yamartino
9.990	maximum error (°)	0.004	−0.092	−0.017
	minimum error (°)	−0.007	−0.096	−0.019
	MAE (mean absolute error) (°)	0.005	0.094	0.018
	% error (of the MAE) (%)	0.05	0.9	0.2
19.968	maximum error (°)	0.053	−0.594	−0.077
	minimum error (°)	−0.031	−0.622	−0.097
	MAE (mean absolute error) (°)	0.024	0.612	0.091
	% error (of the MAE) (%)	0.1	3.1	0.5
29.960	maximum error (°)	0.285	−1.446	−0.125
	minimum error (°)	0.010	−1.531	−0.196
	MAE (mean absolute error) (°)	0.104	1.500	0.172
	% error (of the MAE) (%)	0.3	5.0	0.6
49.886	maximum error (°)	2.911	−2.694	−0.145
	minimum error (°)	1.665	−2.993	−.476
	MAE (mean absolute error) (°)	2.139	2.876	0.355
	% error (of the MAE) (%)	4.3	5.8	0.7
76.543	maximum error (°)	31.759	−4.909	−0.480
	minimum error (°)	25.067	−5.703	−1.155
	MAE (mean absolute error) (°)	27.952	5.452	0.819
	% error (of the MAE) (%)	36.5	7.1	1.0
101.685	maximum error (°)	2617.153	−13.004	−0.255
	minimum error (°)	628.747	−14.471	−0.747
	MAE (mean absolute error) (°)	1516.153	13.741	0.489
	% error (of the MAE) (°)	1491.	13.5	0.5

5. Conclusion

The method of estimation of σ_a by Yamartino (1984) which requires only three accumulators over a given time period and seems readily adaptable to on-line data processing gives excellent results over the entire range of possible standard deviations 0° to 103.9°. (Here, 102.6 was tested.) The approximation of the mean wind direction by the arctangent of the mean sines and mean cosines also gives excellent results; errors are <1° for distributions with $\sigma_a < 75°$. Distributions having $\sigma_a > 75°$ are approaching circular distributions and for these, mean wind direction has little meaning.

Acknowledgments. The author is grateful for the existence of the NOAA Boulder Atmospheric Observatory Tower and the care taken by the individuals maintaining that facility and checking the data from it. The assistance of Jim Wilczak in furnishing wind direction data sets from the NOAA BOA Tower is appreciated. The prompt reply by Messrs. Verrall and Williams to my inquiry, resulting in the addition of the absolute value notations to their equations is also appreciated. The secretarial assistance of Sylvia Coltrane is gratefully acknowledged.

APPENDIX

FORTRAN Subroutines for Three Sigma Methods

In the main program accumulators are initialized:

```
      AN = 0.
      SS = 0.
      SC = 0.
      SSS = 0.
      SCC = 0.
      SSC = 0.
C     AN IS THE NUMBER OF VALUES
C     SS IS THE SUM OF SINES
C     SC IS THE SUM OF COSINES
C     SSS IS THE SUM OF SINES SQUARED
C     SCC IS THE SUM OF COSINES SQUARED
C     SSC IS THE SUM OF SINE TIMES COSINE
```

As each instantaneous value of wind direction A is obtained, the following is accomplished:

```
      AN = AN + 1.
C       CONVERT ANGLE TO RADIANS
      A = A/57.29578
      S = SIN(A)
      C = COS(A)
      SS = SS + S
```

```
SC = SC + C
SSS = SSS + S * S
SCC = SCC + C * C
SSC = SSC + S * C
```

At the conclusion of obtaining all instantaneous values over the time interval, p, the mean wind direction is approximated using the function:

```
ABAR = ARES(SS,SC)
```

and the following calls are made to the methods:

```
CALL ACKER(AN,SS,SC,SSS,SCC,SSC,
  SIGD)
CALL VERWIL(AN,SS,SC,SSS,SCC,
  SIGA)
CALL YATINO(AN,SS,SC,SIGTH)
```

The σ_a is SIGD, SIGA, and SIGTH from the Ackermann, Verrall–Williams, and Yamartino methods respectively.

The function for determining mean direction is

```
FUNCTION ARES(SS,SC)
ARES = ATAN2(SS,SC)
ARES = 57.29578 * ARES
IF(ARES.LT.O.) ARES = ARES + 360.
RETURN
END
```

The subroutines are

```
SUBROUTINE ACKER(AN,SS,SC,SSS,
  SCC,SSC,SIGD)
AS = SS/AN
AS2 = AS * AS
AC = SC/AN
AC2 = AC * AC
VARS = SSS/AN - AS2
VARC = SCC/AN - AC2
COVSC = SSC/AN - AS * AC
TSC = 2. * AS * AC * COVSC
SBAR = SQRT(AS2 + AC2)
SIGD = SQRT(AC2 * VARS + AS2 *
  VARC - TSC)/(SBAR * SBAR)
SIGD = 57.29578 * SIGD
RETURN
END

SUBROUTINE VERWIL(AN,SS,SC,SSS,
  SCC,SIGA)
AS = SS/AN
C = SS * SS/AN
SIGS = SQRT((SSS - C)/AN)
AC = SC/AN
D = SC * SC/AN
SIGC = SQRT((SCC - D)/AN)
AS = ABS(AS)
AC = ABS(AC)
VOS = AS - SIGS
VOC = AC + SIGC
VTS = AS + SIGS
VTC = AC - SIGC
DOT = VOS * VTS + VOC * VTC
AO = SQRT(VOS * VOS + VOC * VOC)
AT = SQRT(VTS * VTS + VTC * VTC)
SIGA = 0.5 * ACOS(DOT/(AO * AT))
SIGA = 57.29578 * SIGA
RETURN
END

SUBROUTINE YATINO(AN,SS,SC,
  SIGTH)
AS = SS/AN
AC = SC/AN
E = SQRT(1. - (AS * AS + AC * AC))
SIGTH = ASIN(E) * (1. + 0.1547 * E ** 3.)
SIGTH = 57.29578 * SIGTH
RETURN
END
```

REFERENCES

Ackermann, G. R., 1983: Means and standard deviations of horizontal wind components. *J. Climate Appl. Meteor.*, **22,** 959–961.

Fisher, N. I., 1983: Comment on "A method for estimating the standard deviation of wind directions." *J. Climate Appl. Meteor.*, **22,** 1971.

International Mathematical and Statistical Libraries, 1982: IMSL Library, Edition 9, Reference Manual. IMSL, Inc. Houston TX. 4 volumes.

Irwin, J. S., 1980: Dispersion Estimate Suggestion 9: Processing of Wind Data. (document for internal distribution) Environmental Application Branch, Meteorology and Assessment Division, ESRL, U.S. Environmental Protection Agency, Research Triangle Park, NC.

——, 1983: Estimating plume dispersion—A comparison of several sigma schemes. *J. Climate Appl. Meteor.*, **22,** 92–114.

Kaimal, J. C., and J. E. Gaynor, 1983: The Boulder Atmospheric Observatory. *J. Climate Appl. Meteor.*, **22,** 863–880.

Mardia, K. V., 1972: *Statistics of Directional Data.* Academic Press, London, 357 pp.

Nelson, E. W., 1984: A simple and accurate method for calculation of the standard deviation of the horizontal wind direction. *J. Air Pollut. Control Assoc.*, **34,** 1139–1140.

Turner, D. B., 1979: Atmospheric dispersion modeling. A critical review. *J. Air Pollut. Control Assoc.*, **29,** 502–519.

Verrall, K. A., and R. L. Williams, 1982: A method for estimating the standard deviation of wind direction. *J. Appl. Meteor.*, **21,** 1922–1925.

Yamartino, R. J., 1984: A comparison of several "single-pass" estimators of the standard deviation of wind direction. *J. Climate Appl. Meteor.*, **23,** 1362–1366.

NOTICE

This document has been reviewed in accordance with U.S. Environmental Protection Agency policy and approved for publication. Mention of trade names or commercial products does not constitute endorsement or recommendation for use.

This article documents research that was conducted under the auspices of the U.S. Environmental Protection Agency (USEPA). As an arm of the government, the USEPA has a paid up, non-exclusive, irrevocable, worldwide license to reproduce, prepare derivative works, and distribute to the public all of the material contained herein, and to grant others acting on its behalf the right to do so. The USEPA grants to the National Technical Information Service, Department of Commerce, the right to act as its agent in the distribution of this material.